

Entropic force law in the presence of a noncommutative inspired spacetime for a solar system scale

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Abstract

We first study some aspects of a physically inspired kind of a noncommutative spherically symmetric spacetime based on the Gaussian-smeared mass distribution for a solar system scale. This leads to the elimination of a singularity appeared in the origin of the spacetime. Afterwards, we investigate some features of Verlinde's scenario in the presence of the mentioned spacetime and derive several quantities such as Unruh-Verlinde temperature, the energy and the entropic force on three different types of holographic screens, namely the static, the stretched horizon and the accelerating surface.

Keywords: Entropic Force; Holographic Screens; Noncommutative Geometry.

1 Introduction

It is obvious that there is a connection between gravity and thermodynamics. In 1995, Jacobson showed that the Einstein field equations of general relativity are derived from the first law of thermodynamics [1]. In 2009, Padmanabhan employed the equipartition law of energy and the holographic principle to provide a thermodynamic interpretation of gravity [2] (to review new insights into thermodynamical aspects of gravity see [3] and the references therein). Soon after, Verlinde proposed a new idea to explain the gravity as an entropic force caused by changes in the information associated with the position of massive particles [4]. Verlinde's conjecture has extensively discussed in various theoretical frameworks such as loop quantum gravity [5], modified gravity [6], black hole physics [7], noncommutative geometries [8], cosmological setups [9], braneworld scenarios [10], Friedmann's equations with noncommutativity corrections [11] and other fields [12].

It is widely believed that the entropy incorporates the emergent view of gravity with the fundamental microstructure of a quantum spacetime. Hence, if the origin of gravity is an entropic force, it is required to take into account the microscopic scale effects by using accurate tools such as noncommutative geometry (NCG) to illustrate the microscopic structure of a quantum spacetime, in Verlinde's scenario. The NCG inspired metrics are a class of solutions of Einstein equations which include influences of quantum gravity in very short distances [13]. An easy way to explain this inspired type of NCG theory is to evaluate the mean position of an object by averaging coordinate

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operators on appropriate coordinate coherent states [14]. In this approach, it has been shown that the mean position of a point-like object in a noncommutative manifold is no longer modeled by a Dirac-delta function distribution, but will be displayed by a Gaussian distribution of minimal width $\sqrt{\theta}$, where θ is the smallest fundamental unit of an observable area in the noncommutative coordinates, beyond which coordinate resolution is obscure. Indeed, the emergence of extreme energies at very short distances in a noncommutative manifold yields the strong quantum fluctuations which stop any measurements to observe a particle position with a precision more than an inherent length scale. This leads to the regular behavior at the origin of black holes. As a result, a minimal length induced by averaging noncommutative coordinate fluctuations is appeared in this well-behaved theory of NCG. In other words, the small scale behavior of point-like structures is cured such that the particle mass M , instead of being totally localized at a point, is distributed throughout a region of linear size $\sqrt{\theta}$ as a smeared-like particle (to study some features of black hole thermodynamics by using the NCG inspired model, see [15]).

On the other hand, the failure of general relativity at the scales such as solar system has evidently been observed [16], which indicates the appearance of dark matter to make compatible observational probes with the theory [17]. The important problem of dark energy and dark matter is connected to their nature because they behave like ad hoc gravity sources but in a well-behaved theory of gravity. To avoid this situation and in order to study the entropic force, we utilize a line element derived from $f(R)$ gravity theory ¹ for the solar system scale [19] (see also [20]). We include the noncommutativity correction in the metric derived in [19] and find the entropic force. To this purpose, we use three various kinds of holographic screens, placing at equipotential surfaces, of a noncommutative inspired spacetime for a solar system scale: the static holographic screen, stretched horizon and the accelerating surface. Here it should be noted that for a static 4D metric, the generic results were obtained by Myung and Kim in Ref. [21] and also by Sakalli in Ref. [22]. However, the aim of this paper is to gain insights into the noncommutative inspired spacetime in the solar system scale. Throughout the paper, natural units are used, i.e. $\hbar = c = G = k_B = 1$, and Greek indices run from 0 to 3.

2 Noncommutative inspired spacetime for the solar system

As a fundamental picture of a quantum spacetime, the noncommutativity is applied to exhibit the fuzziness of the spacetime via the commutation relation

$$[x^A, x^B] = i\theta^{AB}, \quad (1)$$

with a parameter θ which measures the amount of the coordinate noncommutativity in the coordinate coherent states approach [14]. In the simplest case, θ^{AB} is an anti-symmetric, real, $D \times D$ matrix (D is the dimension of the spacetime). The physical interpretation of θ^{AB} is the smallest fundamental unit of an observable area in the AB -plane, in the same way as Planck constant \hbar interprets the smallest fundamental unit of an observable phase space in quantum mechanics. Consequently, the resulting geometry is pointless and the concept of the point is no longer meaningful.

The method we choose here is to look for a static, asymptotically flat, spherically symmetric, minimal width, Gaussian distribution of mass whose noncommutative size is governed by the

¹We assume that a picture of equilibrium thermodynamics does exist for $f(R)$ gravity theory in like manner to that in general relativity [18].

parameter $\sqrt{\theta}$. For this purpose, we choose the mass density as a smeared delta-function

$$\rho_\theta(r) = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right). \quad (2)$$

This result is indeed owing to the fundamental uncertainty encoded in the coordinate commutator (1). The smeared mass distribution M_θ is found to lead to the result

$$M_\theta = \int_0^r \rho_\theta(r) 4\pi r^2 dr = M \left[\mathcal{E}\left(\frac{r}{2\sqrt{\theta}}\right) - \frac{r}{\sqrt{\pi\theta}} e^{-\frac{r^2}{4\theta}} \right], \quad (3)$$

where $\mathcal{E}(n)$ is the Gaussian error function defined as $\mathcal{E}(n) \equiv 2/\sqrt{\pi} \int_0^n e^{-x^2} dx$. In the regime that noncommutative fluctuations are insignificant, $r/\sqrt{\theta} \rightarrow \infty$, the Gaussian error function is equal to one and one recovers the ordinary mass totally localized at a point, i.e. $M_\theta \rightarrow M$. This means that if $\sqrt{\theta}$ is too small, the background geometry is described as a smooth differential manifold and the smeared-like mass descends to the point-like mass. However, in the limit $r \rightarrow \sqrt{\theta}$, the metric deviates predominantly from the standard one and provides new physics at the small scale.

Saffari and Rahvar [19] solved the field equation in the vacuum for the spherically symmetric metric and obtained the dynamics in the solar system and the galactic scales. The line element for the solar system scale up to the first order in α and under the condition $r \ll d$ is given by

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{\alpha}{d}r\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{\alpha}{d}r\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (4)$$

where $d\Omega^2$ is the line element on the 2-dimensional unit sphere. The parameter α is a small dimensionless constant and d is a characteristic length scale in the order of galactic size. Here, it should be emphasized that the metric above is extracted from an ansatz proposed by the authors in Ref. [19] for the derivative of action as a function of distance from the center

$$F(r) = \left(1 + \frac{r}{d}\right)^{-\alpha}. \quad (5)$$

It is clear that the case of $\alpha = 0$ yields the standard Einstein-Hilbert action and therefore the Schwarzschild metric is recovered. As already pointed out above, the condition for the existence of the extra term $(\alpha/d)r$ in the metric (4) is $r \ll d$. This means that if $r \gg d$ or $r \rightarrow \infty$, then the extra term does not exist and this leads to an asymptotically flat spacetime.

Since the extra term $(\alpha/d)r$ is extremely small, therefore in order to find the noncommutative inspired line element associated with smeared mass sources, with a good approximation, one can simply plug the explicit form for the smeared mass distribution into the metric (4) as follows:

$$ds^2 = -\left(1 - \frac{2M_\theta}{r} + \frac{\alpha}{d}r\right) dt^2 + \left(1 - \frac{2M_\theta}{r} + \frac{\alpha}{d}r\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (6)$$

The line element (6) characterizes the geometry of a noncommutative spherically symmetric spacetime for the solar system scale. Since α/d is very small, the above metric displays a similar behavior to the noncommutative Schwarzschild (NCS) spacetime [13]. Depending on the different values of mass M , the metric displays three possible causal structures. We plot the possibility of having two distinct horizons when the mass is large enough (or $M > M_0$, where M_0 is the minimal nonzero

mass corresponding to the minimal nonzero radius $r_0/\sqrt{\theta}$ ²). This possibility is shown in Fig. 1. As the figure shows, the existence of a minimal nonzero mass and the disappearance of a divergence at the origin are simply seen in the same manner as the NCS black hole. There are two horizons, an inner (or noncommutative) r_i and an outer horizon r_o . In the limit $r/\sqrt{\theta} \rightarrow \infty$, and for $\alpha = 0$, the inner horizon tends to zero, while the outer horizon approaches the Schwarzschild value, $r_o \rightarrow 2M$. It is clear that if α/d is too small, then the curves of g_{00} in the NCS metric (with $\alpha = 0$) and the metric (6) are extremely close to each other. As can be seen from the figure, when α/d changes the minus peak of the curves corresponding to r_0 is almost fixed. However, the distance between horizons decreases with increasing the values of α/d . In other words, as α/d deviates from the zero, the outer horizon decreases but the inner horizon and the minimal nonzero radius remain nearly intact. It is important to note that, for short distances or high energies there is a crucial deviation from the standard Schwarzschild metric. In the next section we use the metric (6) to investigate the entropic picture of gravity in the spacetime under study.

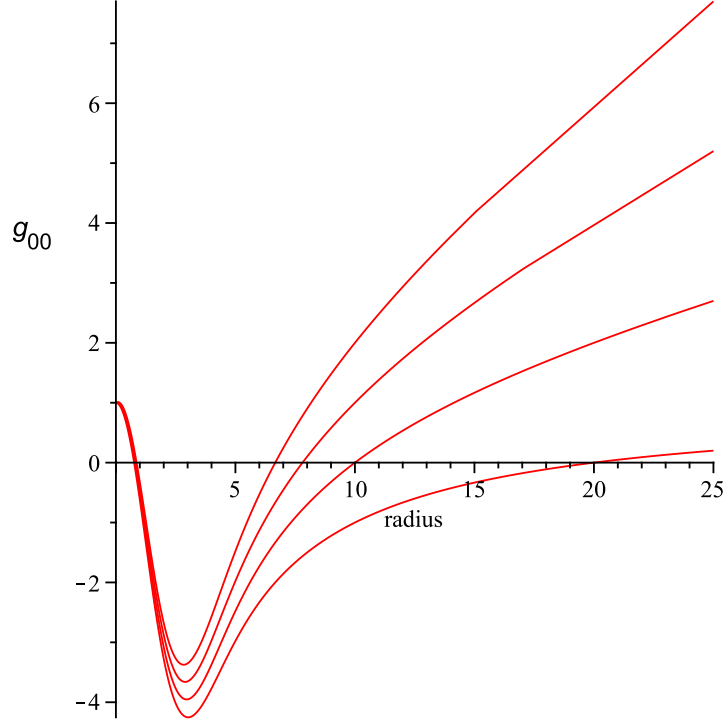


Figure 1: g_{00} versus the radius $r/\sqrt{\theta}$, for different values of α/d . We have set $M = 10.0\sqrt{\theta}$. On the right-hand side of the figure, curves are marked from bottom to top by $\alpha/d = 0$, $0.10/\sqrt{\theta}$, $0.20/\sqrt{\theta}$, and $0.30/\sqrt{\theta}$. This figure shows that, the singularity at the origin is eliminated due to the noncommutativity.

3 Static holographic screen and the entropic force

In order to obtain the entropic force, we should find the timelike Killing vector of the metric (6). Using the Killing equation

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu - 2\Gamma_{\mu\nu}^\lambda \xi_\lambda = 0, \quad (7)$$

²The other possibilities, i.e. $M \leq M_0$, provide less good agreement with the usual physical sense [23].

with the condition of static spherically symmetric $\partial_0 \xi_\mu = \partial_3 \xi_\mu = 0$, and also the infinity condition $\xi_\mu \xi^\mu = -1$, the timelike Killing vector is written as

$$\xi_\mu = \left(\frac{2M_\theta}{r} - \frac{\alpha}{d} r - 1, 0, 0, 0 \right). \quad (8)$$

To define a foliation of space, and distinguishing the holographic screens Ω at surfaces of constant redshift, we write the generalized Newtonian potential ϕ in the general relativistic framework

$$\phi = \frac{1}{2} \log(-g^{\mu\nu} \xi_\mu \xi_\nu), \quad (9)$$

where e^ϕ is the redshift factor and is equal to one at the reference point with $\phi = 0$ at infinity. Hence, the acceleration a^μ is given by

$$a^\mu = -g^{\mu\nu} \nabla_\nu \phi = (0, 2\pi T, 0, 0). \quad (10)$$

The temperature T on the holographic screen is given by Unruh-Verlinde temperature connected to the proper acceleration of a particle close to the screen which can be written as [4]

$$T = -\frac{1}{2\pi} e^\phi n^\mu a_\mu = \frac{e^\phi}{2\pi} \sqrt{g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi}, \quad (11)$$

where $n^\mu = \nabla^\mu \phi / \sqrt{g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi}$ is a unit vector which is normal to the holographic screen and to ξ_μ . The Unruh-Verlinde temperature for the metric (6) is simply achieved and reads

$$T = \frac{1}{2\pi} \left(\frac{M_\theta}{r^2} - \frac{Mr}{2\sqrt{\pi\theta^3}} e^{-\frac{r^2}{4\theta}} + \frac{\alpha}{2d} \right). \quad (12)$$

The energy on the holographic screen Ω for a spherically symmetric spacetime becomes

$$E = \frac{1}{4\pi} \int_\Omega e^\phi \nabla \phi dA, \quad (13)$$

where $A = 4\pi r^2$ is the area of the screen. Hence, the equipartition of energy rule is given by

$$E = \frac{1}{2} AT = 2ST, \quad (14)$$

where $S = A/4$ is the entropy of the holographic screen. Using Eq. (12), the energy on the screen then takes the form

$$E = M_\theta - \frac{Mr^3}{2\sqrt{\pi\theta^3}} e^{-\frac{r^2}{4\theta}} + \frac{\alpha}{2d} r^2. \quad (15)$$

The entropic force on the static holographic screen is found to be

$$F_\mu = T \nabla_\mu S, \quad (16)$$

where $\nabla_\mu S = -2\pi m n_\mu$ is the change in entropy for the test mass m at a fixed place close to the screen. Finally, the entropic force in the presence of the noncommutative spherically symmetric metric for the solar system scale has the form

$$F = \sqrt{g^{\mu\nu} F_\mu F_\nu} = \frac{mM_\theta}{r^2} - \frac{mMr}{2\sqrt{\pi\theta^3}} e^{-\frac{r^2}{4\theta}} + \frac{\alpha m}{2d}. \quad (17)$$

The above result is compatible with the result of Ref. [19]. However, our result is cured in the limit $r \rightarrow 0$. The corresponding relation for the force in Refs. [19, 20] shows a divergence which appears in the origin of the spacetime under study. Utilizing the noncommutativity of coordinates, we eliminate this kind of singularity from the model. The second term at the right hand side of Eq. (17) is a constant force which is independent of the source mass M . In the limit $\theta \rightarrow 0$, and for $\alpha = 0$, the standard results for the acceleration, the local temperature, the energy, and the entropic force on the static holographic screen are recovered, respectively, as follows [24]:

$$a^\mu = \left(0, \frac{M}{r^2}, 0, 0\right), \quad T = \frac{M}{2\pi r^2}, \quad E = M, \quad (18)$$

and

$$F = \frac{mM}{r^2}, \quad (19)$$

which is the correct Newtonian force for the Schwarzschild metric.

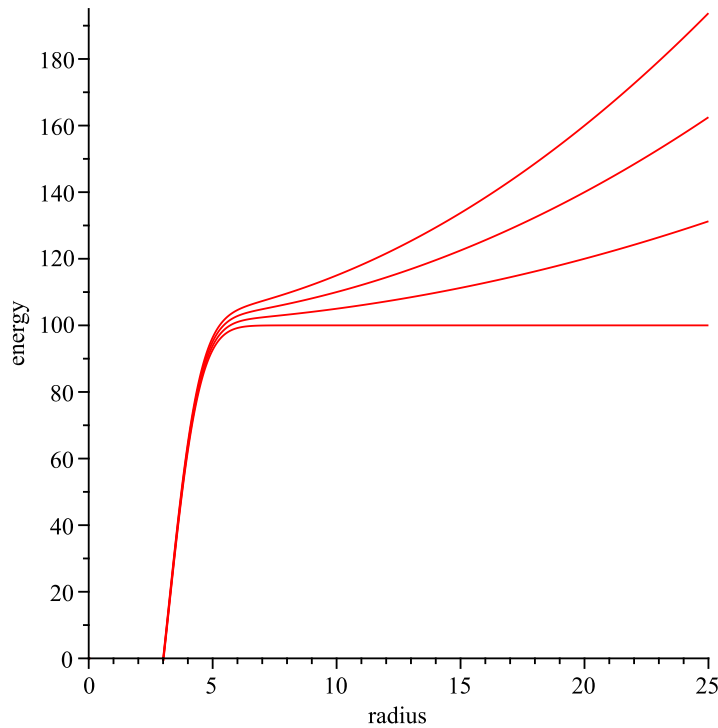


Figure 2: The energy, $E/\sqrt{\theta}$, versus the radius, $r/\sqrt{\theta}$, for different values of α/d . We have set $m = 1.0\sqrt{\theta}$ and $M = 100.0\sqrt{\theta}$. On the right-hand side of the figure, curves are marked from bottom to top by $\alpha/d = 0$, $0.10/\sqrt{\theta}$, $0.20/\sqrt{\theta}$, and $0.30/\sqrt{\theta}$.

The numerical computation of the energy and the entropic force as a function of the radius for some different values of α/d are depicted in Figs. 2 and 3, respectively. Fig. 2 shows the energy increases with deviating the coefficient α/d from the zero. Similarly, as can be seen from Fig. 3, as the coefficient α/d deviates from the zero, the entropic force increases but the peak in the entropic force in the vicinity of the minimal non-zero radius r_0 remains nearly intact. It is worth noting that, these numerical results are fairly similar to the results reported in our previous work [25]. In Ref. [25], the thermodynamical features of a non-commutative inspired Schwarzschild-anti-deSitter

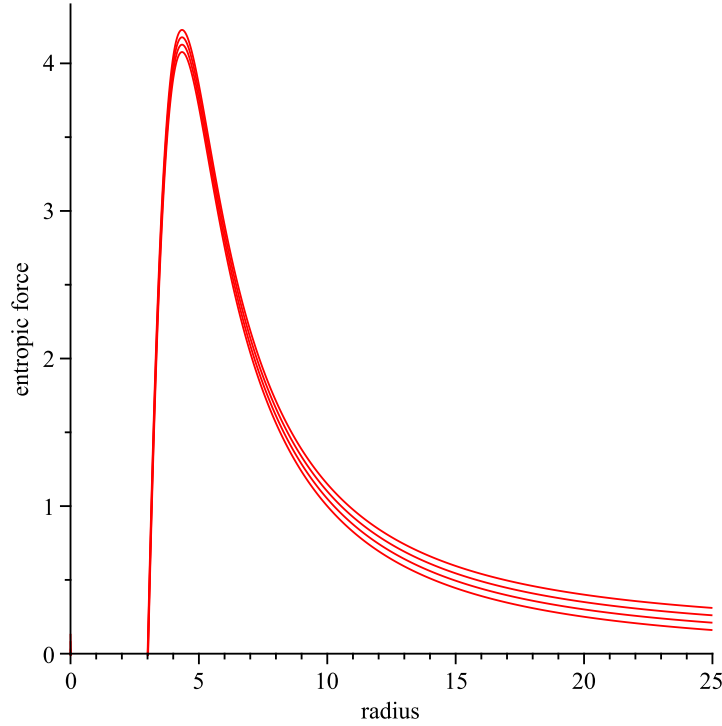


Figure 3: The entropic force F versus the radius, $r/\sqrt{\theta}$, for different values of α/d . We have set $m = 1.0\sqrt{\theta}$ and $M = 100.0\sqrt{\theta}$. On the right-hand side of the figure, curves are marked from bottom to top by $\alpha/d = 0, 0.10/\sqrt{\theta}, 0.20/\sqrt{\theta}$, and $0.30/\sqrt{\theta}$.

black hole in the context of entropic gravity model have been studied. It seems the behavior of the negative cosmological constant and the coefficient α/d associated with the Pioneer anomaly in the solar system scale are the same.

It must be stressed here that the case of $r < r_0$ leads to some out of the standard dynamical features like negative entropic force, i.e. gravitational repulsive force, and negative energy; as a result, one should make the requirement that $E \geq 0$ [23]. Therefore, the appearance of a lower finite cut-off at the short-scale gravity compels a bound on any measurements to determine a particle position in a non-commutative gravity theory.

4 Accelerating surface and the entropic force

In this section, we are interested in introducing the entropic force on the accelerating surface of a noncommutative inspired spacetime for a solar system scale. The accelerating surface was first introduced in [26]. Here, the accelerating surface plays the role of the holographic screen and we consider the accelerating surface as the accelerating screen. By defining a future pointing unit vector u^μ , which is the congruence for the timelike world lines of the points on a spacelike hypersurface S^2 , the orthogonality condition

$$u^\mu n_\mu = 0, \quad (20)$$

should be satisfied, where $n_\mu = [0, 1/\sqrt{-g_{00}}, 0, 0]$ is the normal vector on S^2 . Using the vector u^μ , one can find the change of the heat, which is associated with a proper acceleration vector field as follows:

$$a^\mu = u^\nu \nabla_\nu u^\mu, \quad (21)$$

The only nonzero component of the future pointing unit vector is

$$a^1 = u^0 \nabla_0 u^1. \quad (22)$$

The proper acceleration is found to be

$$a = a^\mu n_\mu = \frac{M_\theta}{r^2} - \frac{Mr}{2\sqrt{\pi\theta^3}} e^{-\frac{r^2}{4\theta}} + \frac{\alpha}{2d}. \quad (23)$$

The Unruh temperature or so-called bulk temperature on the accelerating screen is defined by

$$T = \frac{a}{2\pi}, \quad (24)$$

which means that an accelerating observer on the accelerating screen observes thermal radiation with the Unruh temperature. It is evident that the screen temperature is identical to the bulk temperature and thus its corresponding entropic force on a test mass m in the vicinity of the accelerating screen is equal to Eq. (17).

5 Stretched horizon and the entropic force

In this section, we consider the stretched horizon to be a holographic screen. In this way, the entropic force on the stretched horizon will be obtained. The idea of stretched horizon [27] is interesting due to its location. Its location is extremely adjacent to the event horizon. In other words, all thermodynamical quantities are measured by an observer existing at the proper distance l_{pl} away from the horizon. The radial distance of the stretched horizon from the center of the spacetime under study is determined as $r = r_o + 1/r_o$ such that $r_o \gg 1$. The near horizon limit and the geometry of a stretched horizon can be characterized by a Rindler spacetime. Hence, the local Rindler temperature is immediately written by

$$T = \frac{1}{4\pi} \frac{1}{\sqrt{-g_{00}}} \left| \frac{dg_{00}}{dr} \right| \Bigg|_{r=r_o + \frac{1}{r_o}}. \quad (25)$$

The length contraction of $1/r_o = \sqrt{-g_{00}}$ is as a result of the redshift transformation close to the horizon. Thus, the local temperature on the stretched horizon takes the following form

$$T = \frac{1}{2\pi} \left(\frac{M_\theta(r = r_o + 1/r_o)}{(r_o + 1/r_o)^2} + \frac{\alpha}{2d} \right) - \frac{M(r_o + 1/r_o)}{2\sqrt{\pi\theta^3}} e^{-\frac{(r_o + 1/r_o)^2}{4\theta}} \\ \times \left(1 - \frac{2M_\theta(r = r_o + 1/r_o)}{r_o + 1/r_o} + \frac{\alpha}{d}(r_o + 1/r_o) \right)^{-\frac{1}{2}}. \quad (26)$$

It is important to note that all temperatures on the stretched horizon acquired via different methods are the same in the leading order. So, we have

$$T = \frac{M e^{-\frac{r_o^2}{4\theta}}}{4(\pi\theta)^{\frac{3}{2}}} r_o. \quad (27)$$

In the Rindler space, the local energy yields

$$E = \frac{AMe^{-\frac{r_o^2}{4\theta}}}{8(\pi\theta)^{\frac{3}{2}}}r_o. \quad (28)$$

Finally for the entropic force, we obtain

$$F = \frac{mMe^{-\frac{r_o^2}{4\theta}}}{2\sqrt{\pi\theta^3}}r_o = ma, \quad (29)$$

where $a = \frac{Me^{-\frac{r_o^2}{4\theta}}}{2\sqrt{\pi\theta^3}}r_o$ is the proper acceleration on the stretched horizon. Among three distinct screens we have introduced the stretched one is special due to the fact that it is located at a special location and it is clear that when all the other entropic forces are recomputed for the place of the stretched horizon they all turn into the same [21, 22].

6 Summary

Recently, Verlinde has proposed a new idea of duality between thermodynamics and gravity which yields an emergent phenomenon for the origin of gravity. Since the entropy unites the emergent picture of gravity with the fundamental microstructure of a quantum spacetime, it is therefore essential to take into account the microscopic scale effects via exact tools such as the theory of NCG to clarify the microscopic structure of a quantum spacetime in Verlinde's proposal. For this purpose, we have used the NCG inspired model representing smeared structures to incorporate the microscopic structure of the spacetime with the entropic view of gravity. As a result, the singularity at the origin of the spacetime is removed. We have defined three different surfaces, which are individually such a candidate for the holographic screen of a NCG inspired spacetime. Those are the static holographic screen, the accelerating surface, and the stretched horizon or the Rindler spacetime of the NCG inspired model. In this setup, some thermodynamical quantities of the noncommutative spacetime, e.g. the entropic force for the solar system scale have been derived. We have observed that if one consider the accelerating surface as the accelerating screen, it is very natural to define the acceleration of a test mass through the Unruh temperature and therefore an entropic force can be correctly retrieved. Among these screens, the stretched horizon is particular. Since, the stretched horizon is situated at such a special location, therefore when all the other entropic forces are recomputed for that special place, they all become similar.

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